1980-ÜSS: $10a = \frac{\pi}{2}$ ise $\frac{\cos 4a - \cos 8a}{\cos 4a . \cos 8a} = ?$

Çözüm: $10a = \frac{\pi}{2} \Rightarrow \cos 4a = \sin 6a$ ($4a+6a=10a$)

$\cos 8a = \sin 2a$ ($8a+2a=10a$)

Not: Sinüs ve cosinüsün $90°$ ye tamamlayan açıları birbirine eşittir!.

$$\frac{\sin 6a - \sin 2a}{\cos 4a . \cos 8a} = \frac{2.\sin\left(\frac{6a-2a}{2}\right).\cos\left(\frac{6a+2a}{2}\right)}{\cos 4a . \cos 8a}$$

$$\Rightarrow \frac{2.\sin 2a . \cos 4a}{\cos 4a . \cos 8a} = \boxed{2}$$

Not:

- $\frac{\cos a + \cos 2a + \cos 3a}{\sin a + \sin 2a + \sin 3a} = \cot 2a$

- $\frac{\sin 3a + \sin 7a + \sin 11a}{\cos 3a + \cos 7a + \cos 11a} = \tan 7a$

1988-ÖYS: $\frac{\cos x + \cos 6x + \cos 11x}{\sin x + \sin 6x + \sin 11x} = ?$

Çözüm: Baştaki ve sondaki açıların toplamının yarısı ortadakini veriyorsa, pratik olarak sonuç: $\frac{\cos 6x}{\sin 6x} = \boxed{\cot 6x}$

Örnek: $\frac{1 + \cos 35 + \cos 70}{\sin 35 + \sin 70} = ?$

Çözüm: $\frac{\cos 0 + \cos 35 + \cos 70}{\sin 0 + \sin 35 + \sin 70} = \boxed{\cot 35°}$

Örnek: $\pi = 8x$ ise, $\frac{\cos 13x + \cos 3x}{\sin 7x - \sin 3x} = ?$

Çözüm: $\pi = 8x$ ise $2\pi = 16x$ olur.

* $\cos 13x = \cos 3x$ ($360°$ ye tamamladığından)

$$\frac{\cos 3x + \cos 3x}{2.\sin\left(\frac{7x-3x}{2}\right).\cos\left(\frac{7x+3x}{2}\right)} = \frac{2\cos 3x}{2.\sin 2x . \cos 5x}$$

* $\cos 3x = -\cos 5x$ ($180°$ ye tamamladığından)

* $\sin 2x = \sin 45$

$$\Rightarrow \frac{2.\cos 3x}{2.\sin 45 . \cos 5x} = \frac{-1}{\frac{1}{\sqrt{2}}} = \boxed{-\sqrt{2}}$$

Örnek: $\sin 20 . \sin 40 . \sin 60 . \sin 80 = ?$

Çözüm: $\cos 70 . \cos 50 . \cos 30 . \cos 10$

$\Rightarrow \cos 70 . \cos 10 . \cos 50 . \cos 30$

$\Rightarrow \frac{\cos 80 + \cos 60}{2} . \frac{\cos 80 + \cos 20}{2}$

$\Rightarrow \frac{1}{4}\left[(\cos 80 + \cos 60).(\cos 80 + \cos 20)\right]$

$= \frac{1}{4}\left[(\cos 80 . \cos 80 + \cos 80 . \cos 20 + \cos 60 . \cos 80 + \cos 60 . \cos 20)\right]$

$= \frac{1}{4}\left[\frac{\cos 160 + \cos 0}{2} + \frac{\cos 100 + \cos 60}{2} + \frac{\cos 140 + \cos 20}{2} + \frac{\cos 80 + \cos 40}{2}\right]$

$= \frac{1}{8}\left[-\cos 20 + 1 - \cos 80 + \frac{1}{2} - \cos 40 + \cos 20 + \cos 80 + \cos 40\right]$

$= \frac{1}{8}.\left(1 + \frac{1}{2}\right) = \frac{1}{8}.\frac{3}{2} = \boxed{\frac{3}{16}}$

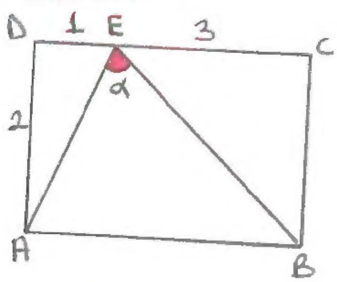
Örnek: $\sin 20 . \sin 40 . \sin 80 = ?$

Çözüm: $\frac{\sin 20 . \sin 40 . \sin 60 . \sin 80}{\sin 60}$

$$= \frac{\frac{3}{16}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{\sqrt{3}}{8}}$$

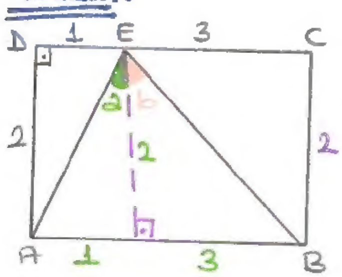
$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$

örnek:



ABCD dikdörtgen ise $\tan\alpha$ kaçtır?

Çözüm:



$\alpha = a + b$ ise

$\tan\alpha = \tan(a+b)$

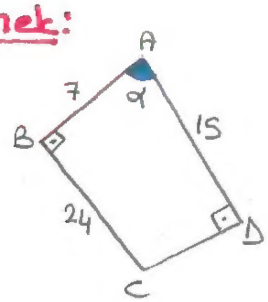
* $\tan a = \dfrac{1}{2}$

* $\tan b = \dfrac{3}{2}$

$$\tan\alpha = \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b} = \frac{\frac{1}{2} + \frac{3}{2}}{1 - \frac{1}{2} \cdot \frac{3}{2}}$$

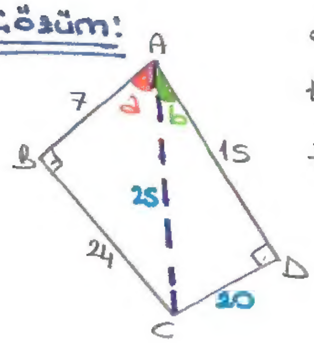
$$\Rightarrow \frac{2}{1 - \frac{3}{4}} = \frac{2}{\frac{1}{4}} = \boxed{8}$$

örnek:



$\tan\alpha$ kaçtır?

Çözüm:



$\alpha = a + b$ ise

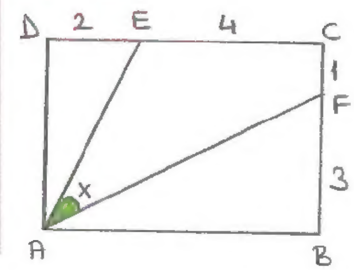
$\tan\alpha = \tan(a+b)$

$\tan a = \dfrac{24}{7}$, $\tan b = \dfrac{20}{15} = \dfrac{4}{3}$

$\tan\alpha = \tan(a+b)$

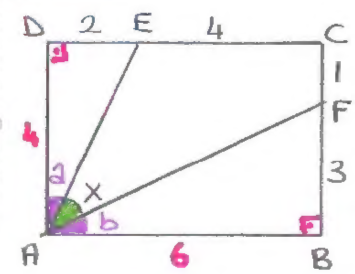
$$\Rightarrow \frac{\frac{24}{7} + \frac{4}{3}}{1 - \frac{24}{7} \cdot \frac{4}{3}} = \boxed{\frac{-4}{3}}$$

örnek:



ABCD dikdörtgen ise $\tan x = ?$

Çözüm:



$x + a + b = 90°$

$x = 90° - (a+b)$

$\tan x = \tan(90 - (a+b))$

$\tan x = \cot(a+b)$

$\tan x = \dfrac{1}{\tan(a+b)}$

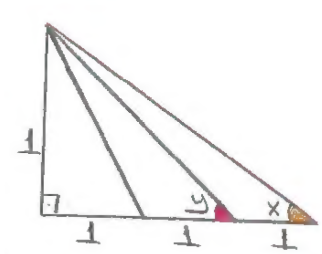
$\tan a = \dfrac{2}{4} = \dfrac{1}{2}$

$\tan b = \dfrac{3}{6} = \dfrac{1}{2}$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b} = \frac{\frac{1}{2} + \frac{1}{2}}{1 - \frac{1}{2} \cdot \frac{1}{2}} = \frac{4}{3}$$

$$\tan x = \frac{1}{\tan(a+b)} = \boxed{\frac{3}{4}}$$

örnek:



$x + y$ kaçtır?

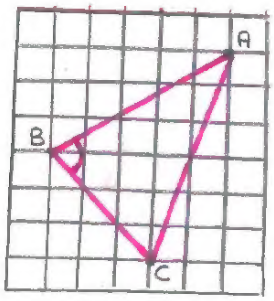
Çözüm:

$\tan x = \dfrac{1}{3}$, $\tan y = \dfrac{1}{2}$

$$\tan(x+y) = \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \cdot \frac{1}{2}} = \frac{\frac{5}{6}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

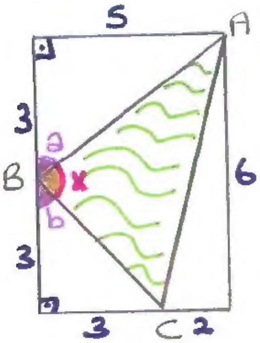
$\tan(x+y) = 1$ ise $\boxed{x + y = 45°}$

2011-LYS:



Birim kareler üzerine çizilmiş yandaki ABC üçgeninin B açısının tanjantı kaçtır?

Çözüm: Şekil, dikdörtgene tamamlanırsa,



$x+a+b=180°$

$x=180°-(a+b)$

$\tan x=\tan(180-(a+b))$

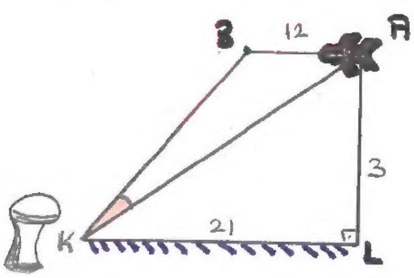
$\tan x=-\tan(a+b)$

- $\tan a=\frac{5}{3}$
- $\tan b=\frac{3}{3}=1$

$\tan(a+b)=\dfrac{\frac{5}{3}+1}{1-\frac{5}{3}\cdot 1}=\dfrac{\frac{8}{3}}{1-\frac{5}{3}}=\dfrac{\frac{8}{3}}{-\frac{2}{3}}=-4$

$\tan x=-\tan(a+b)=--4=\boxed{4}$

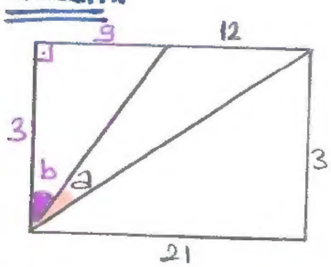
2006-ÖSS:



AB // KL

$\tan(\widehat{AKB})=?$

Çözüm:



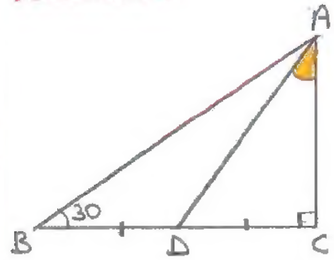
- $\tan(a+b)=\frac{21}{3}=7$
- $\tan b=\frac{9}{3}=3$

$\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a.\tan b}$

$\dfrac{7}{1}=\dfrac{\tan a+3}{1-3.\tan a}$

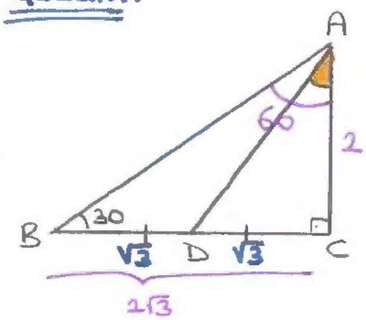
$\boxed{\tan a=\frac{2}{11}}$

1983-ÖYS:



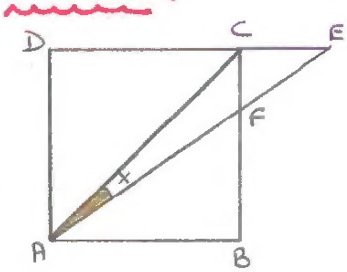
$\tan(\widehat{DAC})=?$

Çözüm:



$\tan(\widehat{DAC})=\frac{\sqrt{3}}{2}$

1978-ÜSS:

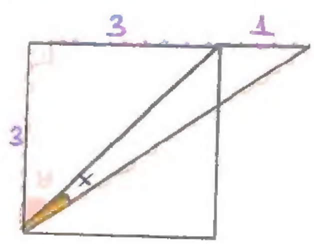


ABCD kare

$|CE|=\frac{|DC|}{3}$ ise

$\tan x=?$

Çözüm:



$\tan(x+y)=\frac{4}{3}$

$\tan y=\frac{3}{3}=1$

$\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x.\tan y}$

$\frac{4}{3}=\dfrac{\tan x+1}{1-1.\tan x}$, $4-4\tan x=3\tan x+3$

$1=7\tan x$, $\boxed{\tan x=\frac{1}{7}}$

cyhnyvz

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Özel olduğunuzu hissedeceksiniz...

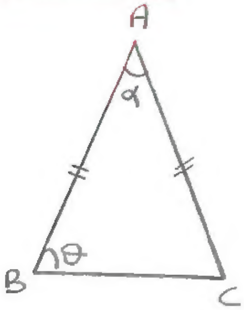
$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$

1970-ÜSS: $\dfrac{\tan 60 - \tan 30}{1 + \tan 60 \cdot \tan 30} = ?$

Çözüm: $\tan(60-30) = \tan 30 = \boxed{\dfrac{\sqrt{3}}{3}}$

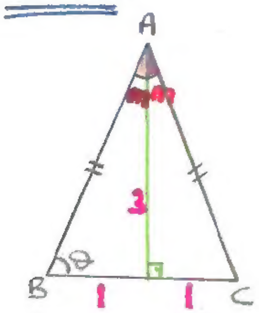
$\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

1997-ÖSS:



$\tan\theta = 3$ ise $\tan\alpha = ?$

Çözüm:



$\alpha = 2m$ olsun.

$\tan\alpha = \tan 2m$

$\tan m = \dfrac{1}{3}$ ise

$\tan\alpha = \tan 2m = \dfrac{2\tan m}{1-\tan^2 m}$

$\tan\alpha = \dfrac{2\cdot\frac{1}{3}}{1-\frac{1}{9}} = \dfrac{\frac{2}{3}}{\frac{8}{9}} = \boxed{\dfrac{3}{4}}$

Uyarı: $a.\cos x \mp b.\sin x$ ifadesinin en büyük ve en küçük değerleri: $\mp\sqrt{a^2+b^2}$

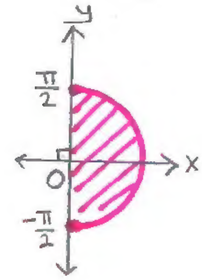
Örnek: $3\sin x - 4\cos x$ ifadesinin alacağı en büyük değer nedir?

Çözüm: $\sqrt{3^2+(-4)^2} = \sqrt{25} = \boxed{5}$

Ters Trigonometrik Fonksiyonlar:

arcsin fonksiyonu:

$\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, 1]$



arcsin fonksiyonu $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ aralığında açı değeri alır, değer aralığı $[-1,1]$ arasındadır.

$y = \sin x \longrightarrow x = \arcsin y$

Yorum: arcsinx demek, hangi açının sinüs değeri x'e eşit demektir. Unutulmamalıdır ki sinüs ve tanjant için x pozitif ise açı 1. bölgede, negatif ise 4. bölgede alınacaktır.

Örnek: $\arcsin\frac{\sqrt{3}}{2} + \arcsin\frac{1}{2} = ?$

Çözüm: $\arcsin\frac{\sqrt{3}}{2} = 60^\circ$, $\arcsin\frac{1}{2} = 30^\circ$

$30^\circ + 60^\circ = \boxed{90^\circ}$

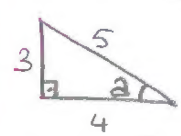
Örnek: $\sin(\arcsin(-1)) = ?$

Çözüm: $\arcsin(-1) = 270^\circ$, $\boxed{\sin 270 = -1}$

Örnek: $\cos\left(\arcsin\frac{3}{5}\right) = ?$

Çözüm: $\arcsin\frac{3}{5} = a$ ise $\sin a = \frac{3}{5}$

$\cos\left(\underbrace{\arcsin\frac{3}{5}}_{a}\right) \longrightarrow \cos a = ?$



$\sin a = \frac{3}{5}$

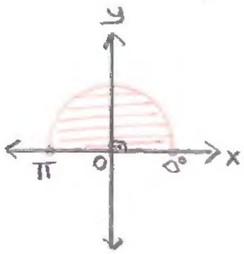
$\boxed{\cos a = \frac{4}{5}}$

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Özel olduğunuzu hissedeceksiniz...

arccos fonksiyonu:

$\cos : [0,\pi] \rightarrow [-1,1]$



arccos fonksiyonu $[0,\pi]$ aralığında açı değeri alır. Değer aralığı da $[-1,1]$ dir.

$x=\cos y \longrightarrow y=\arccos x$

> **Yorum:** arccosx ve arccotx için bulacağımız açı değerleri için x pozitif ise açı 1. bölgede, negatif ise 2. bölgede alınır.

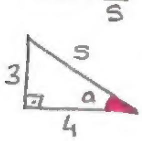
örnek: $\arccos\left(-\frac{1}{2}\right)+\arccos\left(-\frac{\sqrt{2}}{2}\right)=?$

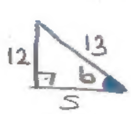
Çözüm: $\arccos\left(-\frac{1}{2}\right)=120^\circ$, $\arccos\left(\frac{-\sqrt{2}}{2}\right)=135^\circ$ $\Big\}$ $120+135=\boxed{255^\circ}$

örnek: $\sin\left(\arccos\frac{4}{5}+\arccos\frac{5}{13}\right)=?$

Çözüm: $\sin(\underbrace{\arccos\tfrac{4}{5}}_{a}+\underbrace{\arccos\tfrac{5}{13}}_{b})=\sin(a+b)=?$

$\cos a=\frac{4}{5}$ $\qquad$ $\cos b=\frac{5}{13}$



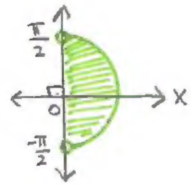


$\sin(a+b)=\sin a.\cos b+\sin b.\cos a$

$=\frac{3}{5}.\frac{5}{13}+\frac{12}{13}.\frac{4}{5}=\frac{15+48}{65}=\boxed{\frac{63}{65}}$

arctan fonksiyonu:

$\tan : \left(-\frac{\pi}{2},\frac{\pi}{2}\right)\rightarrow \mathbb{R}$



$x=\arctan y$

$\tan x=y$

örnek: $4\arctan 1-3\arctan\sqrt{3}=?$

Çözüm: $\arctan 1=45^\circ$

$\arctan\sqrt{3}=60^\circ$

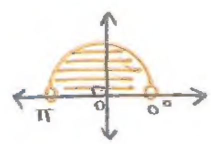
$4\arctan 1-3\arctan\sqrt{3}=4.45-3.60=\boxed{0}$

örnek: $\arctan(-1)+\arctan\left(-\frac{1}{\sqrt{3}}\right)=?$

Çözüm: $\arctan(-1)=315^\circ$, $\arctan\left(-\frac{1}{\sqrt{3}}\right)=330^\circ$ $\Big\}$ $315+330=\boxed{645^\circ}$

arccot fonksiyonu:

$\cot : (0,\pi)\rightarrow \mathbb{R}$



$x=\cot y \Rightarrow y=\operatorname{arccot} x$

örnek: $\operatorname{arccot}\sqrt{3}+\operatorname{arccot}1=?$

Çözüm: $\operatorname{arccot}\sqrt{3}=30^\circ$, $\operatorname{arccot}1=45^\circ$ $\Big\}$ $30+45=\boxed{75}$

örnek: $\sin(2\arctan 3)=?$

Çözüm: $\sin(2\underbrace{\arctan 3}_{a})=\sin(2a)=?$

$\arctan 3=a$

$\tan a=3$



$\sin 2a=2.\sin a.\cos a$

$\sin 2a=2.\frac{3}{\sqrt{10}}.\frac{1}{\sqrt{10}}=\frac{6}{10}=\boxed{\frac{3}{5}}$

#cyhnyuz#

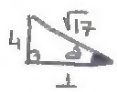
Örnek: $\text{arccot}\frac{1}{4} - \arctan\frac{1}{4} = \arccos x$ ise x kaçtır?

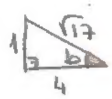
Çözüm: $\underbrace{\text{arccot}\frac{1}{4}}_{a} - \underbrace{\arctan\frac{1}{4}}_{b} = \arccos x$

$a - b = \arccos x \longrightarrow x = \cos(a-b)$

$\text{arccot}\frac{1}{4} = a \qquad \arctan\frac{1}{4} = b$

$\cot a = \frac{1}{4} \qquad \tan b = \frac{1}{4}$





$x = \cos(a-b) = \cos a.\cos b + \sin a.\sin b$

$x = \frac{1}{\sqrt{17}} \cdot \frac{4}{\sqrt{17}} + \frac{4}{\sqrt{17}} \cdot \frac{1}{\sqrt{17}} = \frac{4}{17} + \frac{4}{17} = \boxed{\frac{8}{17}}$

2011-LYS: $f(x) = \arcsin\left(\frac{x}{3}+2\right)$ ise $f^{-1}(x)$ nedir?

Çözüm: $f^{-1}\big(\underbrace{\arcsin\left(\tfrac{x}{3}+2\right)}_{y}\big) = x$

$\arcsin\left(\frac{x}{3}+2\right) = y$ ise $\sin y = \frac{x}{3} + 2$

$\frac{x}{3} = \sin y - 2 \quad , \quad x = 3\sin y - 6$

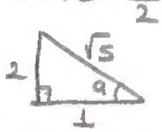
$f^{-1}(y) = 3\sin y - 6 \rightarrow \boxed{f^{-1}(x) = 3\sin x - 6}$

1995-ÖYS: $\cos\left(2\text{arccot}\frac{1}{2}\right) = ?$

Çözüm: $\cos\big(2\underbrace{\text{arccot}\tfrac{1}{2}}_{a}\big) = \cos 2a = ?$

$\text{arccot}\frac{1}{2} = a$

$\cot a = \frac{1}{2}$



$\cos 2a = \cos^2 a - \sin^2 a$

$= \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$

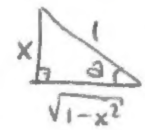
$= \frac{1}{5} - \frac{4}{5} = \boxed{\frac{-3}{5}}$

1976-ÜSS: $\cos(\arcsin x) = ?$

Çözüm: $\cos(\underbrace{\arcsin x}_{a}) = \cos a = ?$

$\arcsin x = a$

$\sin a = x$



$\boxed{\cos a = \sqrt{1-x^2}}$

#cyhnyvz#